

Ch 4: Congruent Triangles

4-1 Congruent Figures

4-2 Triangle Congruence by SSS and SAS

4-3 Triangle Congruence by ASA and AAS

4-4 Using Congruent Triangles: CPCTC

4-5 Isosceles and Equilateral Triangles

4-6 Congruence in Right Triangles

4-7 Using Corresponding Parts of Congruent Triangles

4-1 Congruent Figures:

Focused Learning Target: I will be able to

- Recognize congruent figures and their corresponding parts

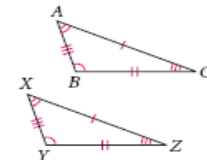
Standards: Geom 4.0
Geom 5.0
Geom. 12.0

Vocabulary:

Congruent Polygon: are polygons that have corresponding sides congruent and corresponding angles congruent.

$$\begin{aligned} \angle A &\cong \angle X & \overline{AB} &\cong \overline{XY} \\ \angle B &\cong \angle Y & \overline{AC} &\cong \overline{XZ} \\ \angle C &\cong \angle Z & \overline{BC} &\cong \overline{YZ} \end{aligned}$$

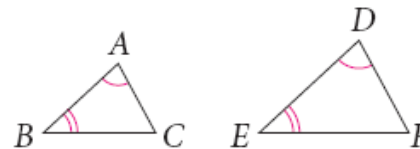
$$\underline{\underline{\triangle ABC \cong \triangle XYZ}}$$



Theorem 4-1

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.

$$\angle C \cong \angle F$$



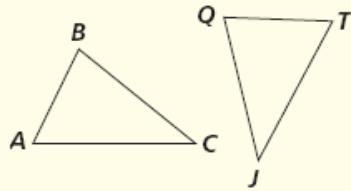
Example 1: Naming Congruent Parts

I'll do one:

We'll do one together:

You try:

1 $\triangle ABC \cong \triangle QTJ$. List the congruent corresponding parts.



$\triangle WYS \cong \triangle MKV$. List the congruent corresponding parts. Use three letters to name each angle.

Draw a picture to represent $\triangle WYS \cong \triangle MKV$. Name all of the pairs of corresponding congruent parts.

Example 2: Find the measure of the angle or the length of the side

I'll do one:

We'll do one together:

You try:

$\triangle WYS \cong \triangle MKV$, $m\angle Y = 67$, & $m\angle M = 48$. Find $m\angle X$

$WASH \cong NOTE$
 $m\angle O = m\angle T = 48$ and $m\angle H = m\angle N$

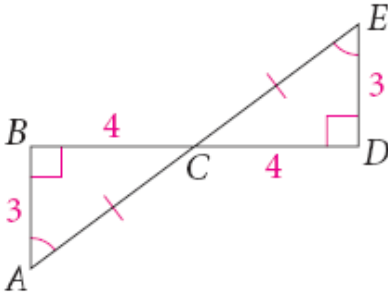
Given: $\triangle WYS \cong \triangle MKV$. If $m\angle Y = 35$, what is $m\angle K$? Explain.

You Try:

Example 3: Finding Congruent Triangles:

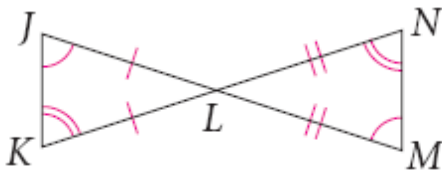
I'll do one:

Decide whether the triangles are congruent. Justify your answer



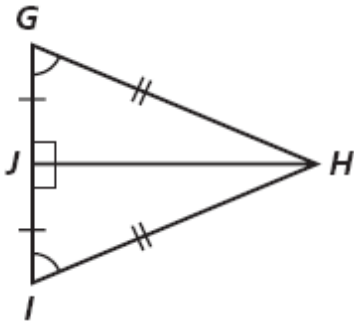
I'll do one more:

Decide whether the triangles are congruent. Justify your answer



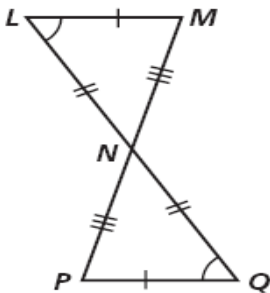
We'll do one together:

Decide whether the triangles are congruent. Justify your answer



You Try:

Decide whether the triangles are congruent. Justify your answer

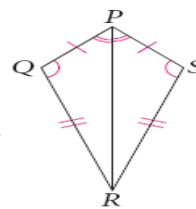


Example 4: Proving Triangles Congruent:

I'll do one:

Given: $\overline{PQ} \cong \overline{PS}$, $\overline{QR} \cong \overline{SR}$, $\angle Q \cong \angle S$, $\angle QPR \cong \angle SPR$

Prove: $\triangle PQR \cong \triangle PSR$



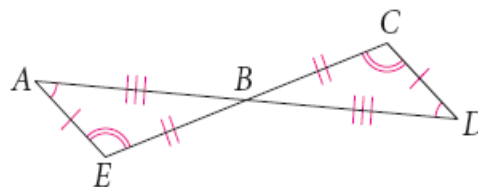
Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

We'll do one together:

Given: $\angle A \cong \angle D$, $\angle E \cong \angle C$, $\overline{AE} \cong \overline{DC}$.

$\overline{EB} \cong \overline{CB}$, $\overline{BA} \cong \overline{BD}$

Prove: $\triangle AEB \cong \triangle DCB$

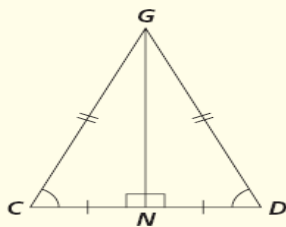


Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

You try:

4 Given: $\overline{CG} \cong \overline{DG}$, $\overline{CN} \cong \overline{DN}$, $\angle C \cong \angle D$, $\triangle CNG$ and $\triangle DNG$ are right triangles.

Prove: $\triangle CNG \cong \triangle DNG$.



Statement	Reason
1.	1.

3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

4-2 Triangle Congruence by SSS and SAS:

Focused Learning Target: I will be able to

- Proving two triangles congruent using the SSS and SAS Postulates

Standards:

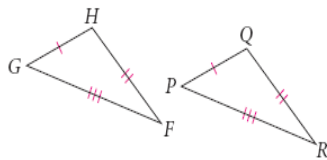
Geom 2.0

Geom 5.0

Postulate 4-1 Side-Side-Side (SSS) Postulate

If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

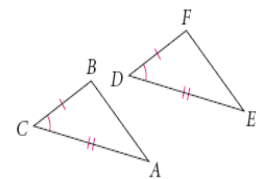
$$\triangle GHF \cong \triangle PQR$$



Postulate 4-2 Side-Angle-Side (SAS) Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

$$\triangle BCA \cong \triangle FDE$$

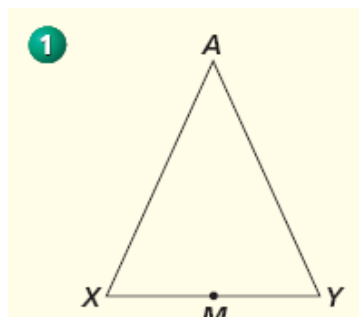


Using SSS:

I'll do one:

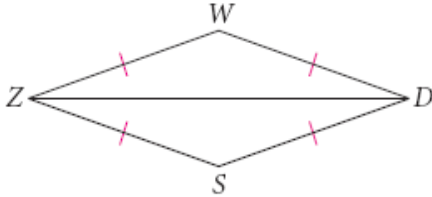
<p>Given: $\overline{HF} \cong \overline{HJ}; \overline{FG} \cong \overline{JK}$ H is midpoint of \overline{GK}.</p> <p>Prove: $\triangle FGH \cong \triangle JKH$</p>	
1.	
2.	
3.	
4.	
5.	
6.	

We'll do one together:




	<p>Given: $\overline{AX} \cong \overline{AY}$; M is midpoint of \overline{XY}. Prove: $\triangle AMX \cong \triangle AMY$</p>	
	1.	
	2.	
	3.	
	4.	
	5.	
	6.	

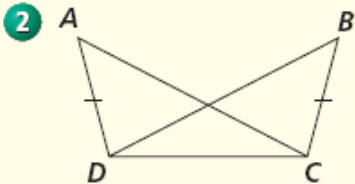
You Try:

	<p>Given: $\overline{WZ} \cong \overline{ZS} \cong \overline{SD} \cong \overline{DW}$ Prove: $\triangle WZD \cong \triangle SDZ$</p>	<p>Given: $\overline{WZ} \cong \overline{ZS} \cong \overline{SD} \cong \overline{DW}$ Prove: $\triangle WZD \cong \triangle SDZ$</p> 
	1.	
	2.	
	3.	
	4.	
	5.	
	6.	

I'll do one:

<p>$\overline{RS} \cong \overline{TK}$. What other information do you need to prove $\triangle RSK \cong \triangle TKS$ by SAS?</p>	
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We'll do one together:

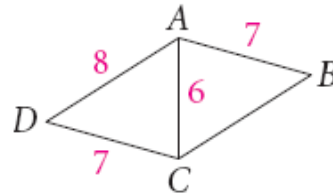


$\overline{AD} \cong \overline{BC}$. What other information do you need to prove

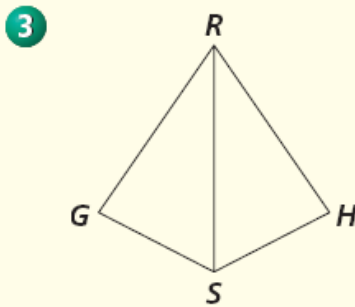
$\triangle ADC \cong \triangle BCD$ by SAS?

You Try:

What other information do you need to prove $\triangle ABC \cong \triangle CDA$ by SAS?



I'll do one

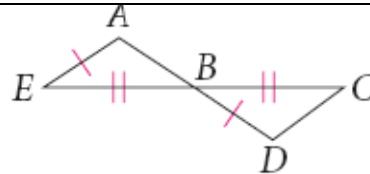


Given: $\angle RSG \cong \angle RSH$, $\overline{SG} \cong \overline{SH}$.
From the information given, can you prove $\triangle RSG \cong \triangle RSH$? Explain.

We'll do one together:

3 From the information given, can you prove $\triangle AEB \cong \triangle DBC$? Explain.

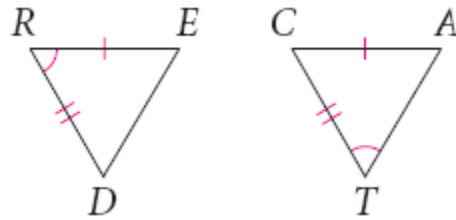
Given: $\overline{EB} \cong \overline{CB}$, $\overline{AE} \cong \overline{DB}$



You Try:

From the information given, can you prove $\triangle RED \cong \triangle CAT$? Explain.

Given: $\overline{RE} \cong \overline{CA}$, $\overline{RD} \cong \overline{CT}$, $\angle R \cong \angle T$



Example 4: Proof:

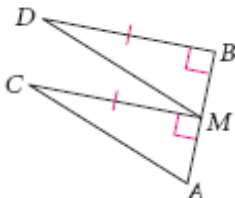
I'll do one:

	<p>Given: \overline{GK} bisects $\angle JGM$, $\overline{GJ} \cong \overline{GM}$.</p> <p>Prove: $\triangle GJK \cong \triangle GMK$</p>	
Statements	Reasons:	
1.	1.	
2.	2.	
3.	3.	
4.	4.	
5.	5.	
6.	6.	

We'll do one together:

	<p>Given: $\overline{FG} \parallel \overline{KL}$, $\overline{FG} \cong \overline{KL}$</p> <p>Prove: $\triangle FGK \cong \triangle KLF$</p>	
Statements	Reasons:	
1.	1.	
2.	2.	
3.	3.	
4.	4.	
5.	5.	
6.	6.	

You try:

<p>Given: $\overline{AB} \perp \overline{CM}, \overline{AB} \perp \overline{DB}, M$ is the midpoint of $\overline{AB}, \overline{CM} \cong \overline{DB}$.</p> <p>Prove: $\triangle AMC \cong \triangle MBD$ See left.</p>	
Statements	Reasons:
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

4-3 Triangle Congruence by ASA and AAS:

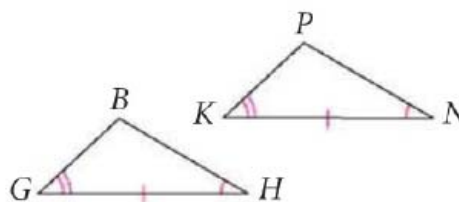
Focused Learning Target: I will be able to

- Prove two triangles congruent using the ASA Postulate and the AAS theorem

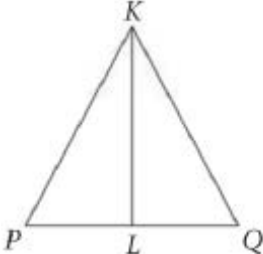
Postulate 4-3 Angle-Side-Angle (ASA) Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

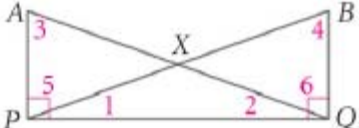
$$\triangle HGB \cong \triangle NKP$$



I'll do one:

<p>Given: $\angle PKL \cong \angle QKL; \overline{KL} \cong \overline{PQ}$</p> <p>Prove: $\triangle PKL \cong \triangle QKL$</p>	
Statements:	Reasons:
1.	1.
2.	2.
3.	3.
4.	4.

We'll do one together:

<p>Given: $\angle A \cong \angle B; \overline{AX} \cong \overline{XQ}$</p> <p>Prove: $\triangle AXP \cong \triangle BXQ$.</p>	
Statements:	Reasons:
1.	1.

2.	2.
3	3.
4.	4.

You try:

Given: $\angle U \cong \angle E; \angle X \cong \angle O; \overline{UX} \cong \overline{EO}$ Prove: $\triangle UXT \cong \triangle EOD$	
Statements:	Reasons:
1.	1.
2.	2.
3.	3.
4.	4.

You Try another:

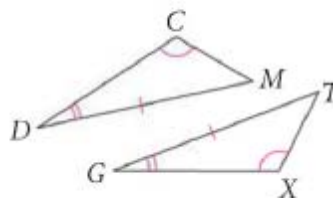
Given: $\angle M \cong \angle P; \overline{MN} \cong \overline{PN}$ Prove: $\triangle LMN \cong \triangle OPN$	
Statements:	Reasons:
1.	1.
2.	2.
3.	3.
4.	4.

Theorem 4-2

Angle-Angle-Side (AAS) Theorem

If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent.

$$\triangle CDM \cong \triangle XGT$$

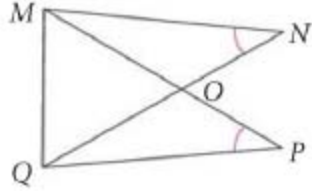


I'll do one:

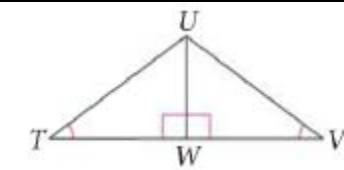
Given: $\angle S \cong \angle Q; \angle SRP \cong \angle QRP$ Prove: $\triangle SRP \cong \triangle QRP$	
Statements:	Reasons:
1.	1.
2.	2.
3.	3.

4	4.
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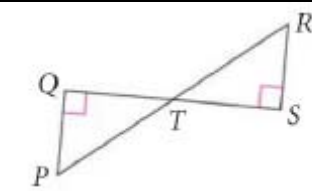
We'll do one:

Given: $\angle N \cong \angle P$; $\overline{MO} \cong \overline{QO}$ Prove: $\triangle NOM \cong \triangle POQ$	
Statements:	Reasons:
1.	1.
2.	2.
3.	3.
4.	4.

You Try:

Given: $\angle T \cong \angle V$; $\angle TWU \cong \angle VWU$ Prove: $\triangle TWU \cong \triangle VWU$	
Statements:	Reasons:
1.	1.
2.	2.
3.	3.
4.	4.

You Try another:

Given: $\overline{PT} \cong \overline{TR}$; $\angle Q$ & $\angle S$ are right angles Prove: $\triangle TWU \cong \triangle VWU$	
Statements:	Reasons:
1.	1.
2.	2.
3.	3.
4.	4.

4-4 Using Congruent Triangles: CPCTC

Focused Learning Target: I will be able to <ul style="list-style-type: none"> Use triangle congruence and CPCTC to prove that parts of two triangles are congruent. 	CA Standard(s): 2.0 & 5.0
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Vocabulary: <ul style="list-style-type: none"> CPCTC

Once you know that triangles are congruent, then you know that the corresponding parts are congruent. You can abbreviate this as CPCTC (Corresponding Parts of Congruent Triangles are Congruent)

Example:

Given: $\triangle JCR \cong \triangle HGV$ Prove: $\overline{CR} \cong \overline{GV}$	
Statement:	Reasons:
1.	1.
2.	2.

I'll do one:

Given: $\angle ABD \cong \angle CBD$; $\angle BDA \cong \angle BDC$ Prove: $\overline{AB} \cong \overline{CB}$	
Statements:	Reasons:
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

We'll do one together:

Given: $\overline{FS} \cong \overline{EJ}$; $\overline{JF} \cong \overline{SV}$; $\overline{EF} \cong \overline{FV}$ Prove: $\angle J \cong \angle S$	23.
Statements:	Reasons:
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

You Try:

Given: $\overline{AC} \cong \overline{CE}$; $\overline{BC} \cong \overline{CD}$ Prove: $\angle B \cong \angle D$	
Statements:	Reasons:
1.	1.
2.	2.
3.	3.
4.	4.

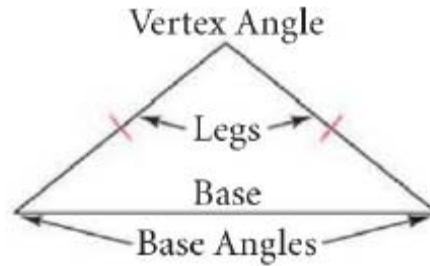
5.	5.
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4-5 Isosceles and Equilateral Triangles

Focused Learning Target: <ul style="list-style-type: none"> To use and apply properties of isosceles triangles 	CA Standard(s): 4.0, 5.0, and 12.0
Vocabulary: <ul style="list-style-type: none"> Legs Base 	<ul style="list-style-type: none"> Vertex angle Base angles corollary

Isosceles triangles have certain parts that we need to identify before learning the properties about them.

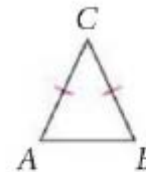
- Legs** – the two congruent sides of an isosceles triangle
- Base** – the third side (doesn't always have to be the bottom side!)
- The Vertex angle** – the angle between the congruent legs
- Base angles** – the other two angles on both ends of the base



Theorem 4-3 Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

$$\angle A \cong \angle B$$



Theorem 4-4 Converse of Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite the angles are congruent.

$$\overline{AC} \cong \overline{BC}$$



Theorem 4-5

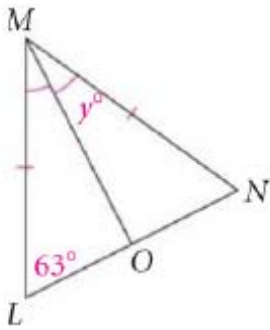
The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base.

$$\overline{CD} \perp \overline{AB} \text{ and } \overline{CD} \text{ bisects } \overline{AB}.$$



I'll do one:

Find the value of y .

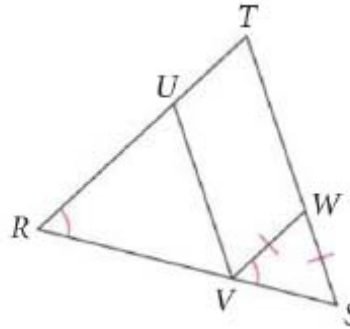


We'll do one together:

Explain/Justify why each statement is true:

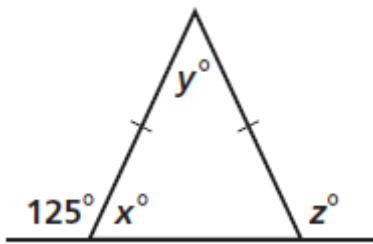
a) $\angle WVS \cong \angle S$

b) $\overline{TR} \cong \overline{TS}$



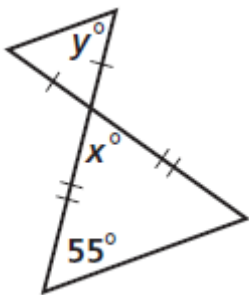
We'll do another one together:

Find the measures of each variable.



You try one:

Find the measures of each variable.



Let's prove the Isosceles Triangle Theorem 4-3

I'll do one:

Given: $\overline{XY} \cong \overline{XZ}$, \overline{XB} bisects $\angle YXZ$

Prove: $\angle Y \cong \angle Z$

Statement	Reason
1. $\overline{XY} \cong \overline{XZ}$	1. Given

	2. \overline{XB} bisects $\angle YXZ$	2. Given
	3.	3.
	4.	4.
	5.	5.
	6.	6.

Let's now prove the Isosceles Triangle Theorem 4-3 in a different way.
We'll do one together:

<p>Given: $\overline{KH} \cong \overline{KJ}$, \overline{KM} bisects \overline{HJ}</p> <p>Prove: $\angle H \cong \angle J$</p>	
Statement	Reason
1. $\overline{KH} \cong \overline{KJ}$	1.
2. \overline{KM} bisects \overline{HJ}	2.
3. $\overline{HM} \cong \overline{JM}$	3.
4. $\overline{KM} \cong \overline{KM}$	4.
5. $\triangle KHM \cong \triangle KJM$	5.
6. $\angle H \cong \angle J$	6.

4-6 Congruence in Right Triangles

<p>Focused Learning Target: I will be able to</p> <ul style="list-style-type: none"> Prove right triangles are congruent using the HL Theorem. 	CA Standard(s): 4.0 & 5.0
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<p>Vocabulary:</p> <ul style="list-style-type: none"> Hypotenuse 	<ul style="list-style-type: none"> Legs
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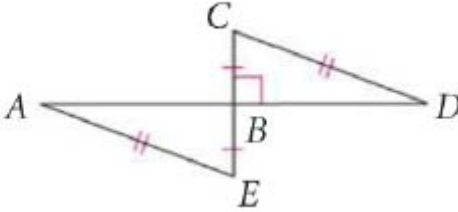
Right triangles have certain parts that we need to identify before learning the properties about them.

<ul style="list-style-type: none"> Hypotenuse – the side opposite the right angle is also the longest side of a right triangle Legs – the other two sides of a right triangle 	
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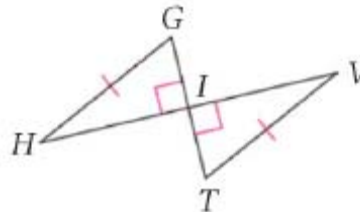
Theorem 4-6 Hypotenuse-Leg (HL) Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

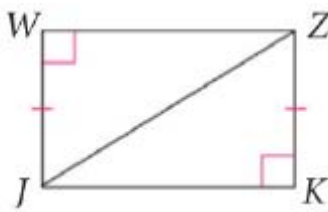
I'll do one:

<p>Given: $\overline{CD} \cong \overline{EA}$, \overline{AD} is the perpendicular bisector of \overline{CE}.</p> <p>Prove: $\triangle CBD \cong \triangle EBA$</p> 	Statement	Reason
	1. $\overline{CD} \cong \overline{EA}$	1. Given
	2. \overline{AD} is the perpendicular bisector of \overline{CE}	2. Given
	3. $\angle CBD$ and $\angle EBA$ are right angles	3.
	4. $\triangle CBD$ and $\triangle EBA$ are right triangles	4.
	5. $\overline{CB} \cong \overline{EB}$	5.
6. $\triangle CBD \cong \triangle EBA$	6.	

We'll do one together:

<p>Given: $\overline{HV} \perp \overline{GT}$, $\overline{GH} \cong \overline{TV}$, I is the midpoint of \overline{HV}.</p> <p>Prove: $\triangle IGH \cong \triangle ITV$</p> 	Statement	Reason
	1. $\overline{HV} \perp \overline{GT}$	1. Given
	2. $\angle GIH$ and $\angle TIV$ are right angles	2.
	3. $\triangle IGH$ and $\triangle ITV$ are right triangles	3.
	4. $\overline{GH} \cong \overline{TV}$	4.
	5. I is the midpoint of \overline{HV}	5.
	6. $\overline{HI} \cong \overline{VI}$	6.
7. $\triangle IGH \cong \triangle ITV$	7.	

You Try one:

<p>Given: $\overline{WJ} \cong \overline{KZ}$, $\angle W$ and $\angle K$ are right angles.</p> <p>Prove: $\triangle JWZ \cong \triangle ZKJ$</p> 	Statement	Reason
	1. $\overline{WJ} \cong \overline{KZ}$	1. Given
	2. $\angle W$ and $\angle K$ are right angles	2. Given
	3. $\triangle JWZ$ and $\triangle ZKJ$ are right triangles	3.
	4. $\overline{JZ} \cong \overline{JZ}$	4.
5. $\triangle JWZ \cong \triangle ZKJ$	5.	

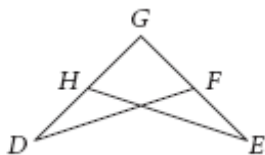
4-7 Using Corresponding Parts of Congruent Triangles:

<p>Focused Learning Target: I will be able to</p> <ul style="list-style-type: none"> Identify congruent overlapping triangles. Prove two triangles congruent by first proving Two other triangles congruent. 	CA Standard(s): 4.0, 5.0, and 12.0
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I'll do one:

Separate and redraw the indicated triangles. Identify any common angles or sides.

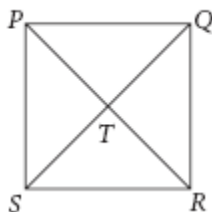
$\triangle DFG$ and $\triangle EHG$



We'll do one together:

Separate and redraw the indicated triangles. Identify any common angles or sides.

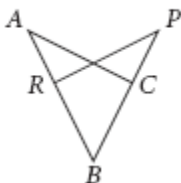
$\triangle PQS$ and $\triangle QPR$



You Try one:

Separate and redraw the indicated triangles. Identify any common angles or sides.

$\triangle ACB$ and $\triangle PRB$



I'll do one:

	<p>Given: $\angle ZXW \cong \angle YWX$, $\angle ZWX \cong \angle YXW$ Prove: $\overline{ZW} \cong \overline{YX}$</p>	
	Statements:	Reasons:
	1. $\angle ZXW \cong \angle YWX$	1.
	2.	2.
	3.	3.
	4.	4.

We'll do one together:

	Given: $\angle T \cong \angle R, \overline{PQ} \cong \overline{PV}$ Prove: $\angle PQT \cong \angle PVR$	
Statements:	Reasons:	
1.	1.	
2.	2.	
3.	3.	
4.	4.	
5.	5.	

You Try one:

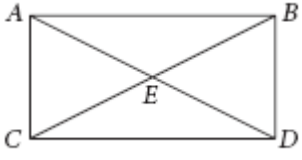
	Given: $\overline{QD} \cong \overline{UA}, \angle QDA \cong \angle UAD$ Prove: $\overline{QA} \cong \overline{UD}$	
Statements:	Reasons:	
1.	1.	
2.	2.	
3.	3.	
4.	4.	
5.	5.	

I'll do one:

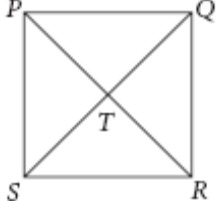
	Given: $\overline{ON} \perp \overline{NP}, \overline{PQ} \perp \overline{QO}, \overline{ON} \cong \overline{PQ}$ Prove: $\triangle PNO \cong \triangle QOP$	
Statements:	Reasons:	
1.	1.	
2.	2.	

	3.	3.
	4.	4.
	5.	5.

We'll do one together:

<p>Given: $\triangle ADC \cong \triangle BCD$ Prove: $\overline{CE} \cong \overline{ED}$</p>	
Statements:	Reasons:
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

You Try one:

<p>Given: $\triangle PRS \cong \triangle QPR$ Prove: $\overline{ST} \cong \overline{TR}$</p>	
Statements:	Reasons:
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.